Chapter 5 Note:

When $s(t)$ is a position function, we know $s'(t)$ is the velocity of the object.

Now, acceleration is $s''(t)$.

Supplement - Day 1

<u>Exponential Notation</u>: if a is any real number and n is a positive integer, then the <u>n^{th} power of a</u> is a^n.

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$$

a is called the <u>base</u>.

n is called the <u>exponent</u>.

<u>Note</u>: $(a^{1/3})^3 = a^{(1/3)(3)} = a^1 = a$

thus $a^{1/3} = \sqrt[3]{a}$

In general, $a^{1/n} = \sqrt[n]{a}$

<u>Rational Exponents</u>: for any rational exponent m/n in lowest terms, where m and n are integers and $n>0$

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

$$a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$$

if n is even, we require $a \geq 0$

Laws of Exponents

$a^0 = 1$

$a^x a^y = a^{x+y}$

$a^{-x} = \frac{1}{a^x}$

$\frac{a^x}{a^y} = a^{x-y}$

$(ab)^x = a^x b^x$

$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

$(a^x)^y = a^{xy}$

Ex: Simplify $(3^5)^9 = 3^{5 \cdot 9} = 3^{45}$

Ex: Simplify $a^3 a^{-6} = a^{3+-6} = a^{-3}$

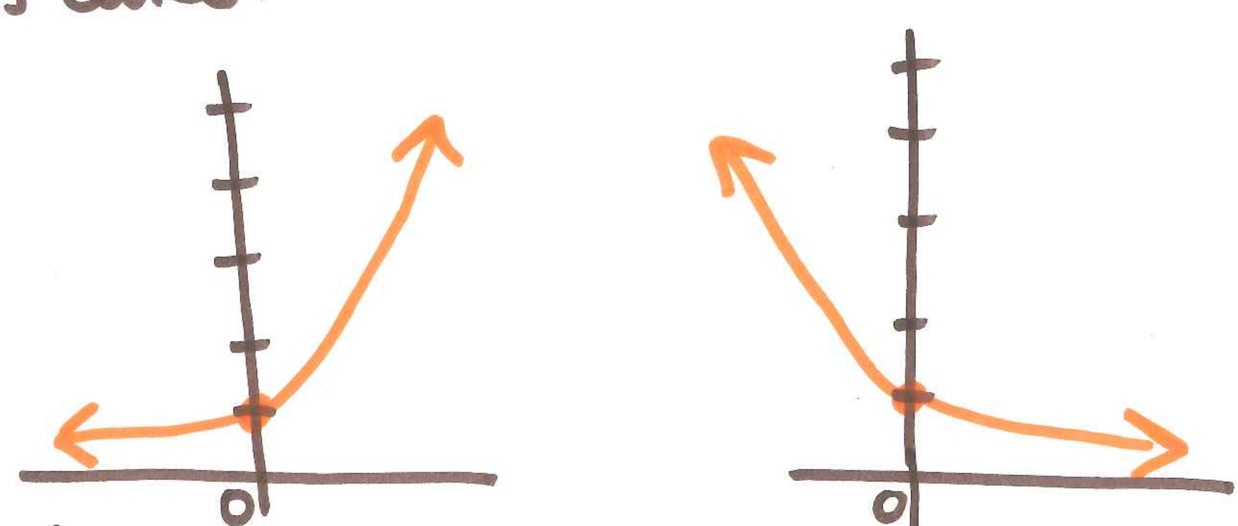
$= \frac{1}{a^3}$

Exponential Functions

Let $a>0$ and $a \neq 1$. The **exponential function with base a** is defined by $f(x)=a^x$ for all real numbers x

Graphs

$f(x)=a^x$ $(a>0, a\neq 1)$ has domain $\mathbb{R}$ and range $(0,\infty)$. The graph looks like:



$a>1$ $\qquad$ $0<a<1$

the most important base for an exponential function is e.

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

e is irrational, but $e \approx 2.71828$

The <u>natural exponent function</u> is

$f(x) = e^x$

Logarithmic Functions

Let $a>0$ and $a \neq 1$. The logarithmic function with base a, denoted $\log_a$, is defined by

$$y = \log_a(x) \iff a^y = x$$

Properties of Logarithms

$\log_a(1) = 0$ $\quad$ $\log_a(a^x) = x$

$\log_a(a) = 1$ $\quad$ $a^{\log_a(x)} = x$

Laws of Logarithms

$$\log_a(AB) = \log_a(A) + \log_a(B)$$

$$\log_a\left(\frac{A}{B}\right) = \log_a(A) - \log_a(B)$$

$$\log_a(A^c) = c\log_a(A)$$

Ex: $\log_5(5^2) = 2$

Ex: $\log_2\left(\frac{2}{2^3}\right) = \log_2(2) - \log_2(2^3)$
$= 1 - 3 = -2$

Change of Base

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

the logarithm with base 10 is called the common log and is denoted by

$$\log(x) = \log_{10}(x)$$

the logarithm with base e is called the <u>natural log</u> and is denoted by

$$\ln(x) = \log_e(x)$$

by our previous definition

$$y = \ln(x) \iff e^y = x$$

<u>Properties of Natural Logs</u>

$\ln(1) = 0$

$\ln(e) = 1$

$\ln(e^x) = x$

$e^{\ln(x)} = x$

<u>Ex</u>: $\ln(e^9) = 9$

Derivatives

$$\frac{d}{dx}(e^x) = e^x \quad \text{OR} \quad (e^x)' = e^x$$

$$\frac{d}{dx}(e^{g(x)}) = e^{g(x)} \cdot \frac{d}{dx}(g(x))$$

$$\text{OR} \quad (e^{g(x)})' = e^{g(x)} \cdot g'(x)$$

Ex: let $f(x) = e^{4x}$

a) find $f'(x)$.

$$f'(x) = e^{4x}(4) = 4e^{4x}$$

b) find $f''(x)$.

$$f''(x) = 4 \cdot e^{4x}(4) = 16e^{4x}$$